**Leaping Black Swans**

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**Abstract**

This article examines an option based portfolio insurance strategy where a fixed percentage of the portfolio is used to purchase in the money long-term call options with the remainder invested in a standard investment grade bond fund. Our results show a 90/10 portfolio, where 10% is invested in long term call options, has returns commensurate with the S&P 500 while mitigating losses. These call option based portfolios don’t require rebalancing during the term of coverage and are flexible enough to suit investors with very different risk tolerances and portfolio sizes. As constructed, these portfolios outperform put option based portfolio insurance strategies and perform as well as a CPPI, with the added advantage of not needing to be actively managed.

**Leaping Black Swans**

A basic tenet of investing is investments with more risk require a higher return. Investors typically manage this risk by holding a diversified portfolio. Significant market declines, such as the dot com crash in 2000 and the recent financial crisis, result in significant loss in portfolio values, even for well diversified portfolios. Investors can use portfolio insurance strategies to mitigate the impact of potential market crashes on the value of investment portfolios. The cost of portfolio insurance is the reduction in gains when the market is rising. Investors must balance the cost of the portfolio insurance against the protection that it provides. How much does portfolio insurance cost? Is there an alternative to the most commonly used methods of portfolio insurance that provides a better combination of decreased risk without sacrificing returns? Is there a simpler model of portfolio insurance that doesn't require frequent rebalancing to provide the protection investors seek?

Historically, the two main strands of PI strategies are option based portfolio insurance (OBPI) strategies detailed in Leland and Rubinstein [1976] and constant proportionate portfolio insurance (CPPI) strategies set forth by Black and Jones [1987]. The cost of protection in OBPI strategies—typically implemented by purchasing a put option— is usually quite high. This cost results in significantly lower returns, on average, than an uninsured portfolio.

CPPI strategies set a floor value of the portfolio and are based on holding a portion of the portfolio in risky assets with the remainder in a risk-free investment. The portfolio is periodically rebalanced between the two asset classes. Market declines trigger purchases in the risk-free asset and sales of the risky portfolio. The rebalancing results in a risk-free investment if the value of the portfolio falls to the floor value, thus eliminating further exposure.

The research on these strategies has been exhaustive—comparing the strategies to each other and relative to a buy-and-hold strategy (see Bertrand and Prigent [2005], Bertrand and Prigent [2011], Dichtl and Drobetz [2011a], Annaert, Osselaer, and Verstraite [2009], and Zieling, Mahayni, and Balder [2014]). Most research finds that OBPI strategies are unable to outperform CPPI strategies (or, for that matter, buy-and-hold benchmarks) due to the costs associated with puts.

However, as Pezier and Scheller [2013] note, most OBPI research evaluates the put option rather than call options.  In the popular press, Bittman [2011] discusses a 90/10 strategy, where 10% of funds are in call options and the remainder is in cash. While Bodie [2001] briefly discusses the payoffs to such a strategy, there is little empirical work regarding a strategy that uses call options, as outlined above, or how it compares to other OBPI or CPPI strategies in terms of the downside risk protection it provides.

This study examines a variation to standard option based portfolio insurance by using a fixed percentage of the portfolio to purchase in the money long-term call options, often referred to as long-term equity anticipation securities (LEAPS), with the remainder invested in a standard investment grade bond fund. This risk mitigation approach, while not entirely new, is largely ignored when comparing PI strategies, as it is considered the same as a synthetic put position. In this study, the call option/bond position differs in three key respects relative to a synthetic put position. First, the percentage in the bond position resets each year to a fixed percentage and is not based on the present value of some strike price. In addition, we use a standard bond portfolio rather than a risk-free bond.  We also purchase in the money LEAPS and sell them before expiration to reduce premium decay.

The proposed portfolio strategy, referred henceforth as a Black Swan (B-S) portfolio, is especially designed to protect from low probability, but high impact events that have the potential to decimate a portfolio and may lead investors to the sidelines for years to come.[1] Furthermore, this downside protection needs to be achieved without giving up the majority of the upside.

One of the differences between traditional PI methods and the B-S portfolio, is the cost. The B-S portfolios require two transactions to set the position for the year. Theoretically, a CPPI portfolio requires constant rebalancing which results in high transaction costs. Black and Jones [1987], in their seminal article on portfolio insurance using CPPI, note the outcomes depend upon how the market does in that year and suggest if volatility is high, portfolio value will approach the floor even if the market is unchanged for the year. The cost of traditional OBPIs, utilizing puts, increases with volatility and during times of distress (when demand for PI is presumably the highest). To the extent the B-S portfolio buys calls, it is not immune to volatility. However, by buying somewhat deep in-the-money (0.7 delta) calls, the time premium (as a percentage of the overall call position) is smaller. Additionally, by selling the calls before expiration, some of the time premium can be recovered if the option remains close to being at-the-money. In summary, this alternative approach to PI is relatively straightforward to initiate, has low transaction costs, and provides flexibility for investors with various risk preferences.

Using data from 1990-2015, findings suggest a 90/10 (90% invested in a bond index & 10% in S&P 500 LEAPS) portfolio has returns commensurate with the S&P 500, while mitigating losses. An 85/15 strategy has returns that exceed the S&P 500, while still reducing maximum loss. Using this type of strategy is robust to the delta of the option, the length of the option, the time periods examined, and even the bond fund used. We affirm the results using

Monte Carlo simulation. The call option PI strategies have significantly better Sortino ratios, value-at-risk (VaR), expected shortfall (ES), Omega values, and greater cumulative prospect value (CPV) relative to buy-and-hold or a protective put. The CPPI strategy appears to fall somewhere between the 95/5 and 90/10 portfolios in terms of risk return results and we find it to be virtually equivalent to a 92/8. As a group, PI strategies can lead to higher long-term returns by preventing large losses due to market crashes. The results indicate that PI strategies are not just appealing from a risk perspective, but also from a long-term return perspective.

## DATA AND METHODOLOGY

This study compares a buy-and-hold strategy to four option based PI portfolios and a CPPI portfolio. We evaluate three PI call option strategies: 85/15, 90/10, and a 95/5, with the first number as the percentage invested in a bond fund and the remaining invested in long-term call options. Call options are selected based on a delta of 0.7 (approximately 6.5% in the money) to reduce the time value (as a percentage of the total price) paid for the option. We purchase 13-month options and sell them with one month remaining to avoid the possible significant time decay that can occur in the expiration month.

Option data on the SPX index is from January 1990-June 2015 acquired from DeltaNeutral whose source is the Options Pricing and Reporting Authority (OPRA). Since long-term options (LEAPS) are generally only available for June and December/January, June and December options are assumed to be purchased at the end of May or the end of November with an average maturity of 385 days. We also investigate purchasing the options with 16 months to maturity and selling with three months remaining. Return data for the S&P 500, Vanguard's investment grade bond fund (VBMFX), and treasuries is from the Center for Research in Security Prices (CRSP).

We calculate yearly returns for the option based on selling the option purchased in the previous year with a remaining average time to maturity of 20 days.[2] At that time, we purchase a new 13-month option. We use the end-of-day ask price to determine the purchase price, and the end-of-day bid price to determine the sell price**.** Thus, the 90/10 portfolio return over the year is 90% of the VBMFX return plus 10% of the option return. If, for example, the option returns 100% for the year, the portfolio will earn 10% from the option position alone.

We compare the B-S portfolios to both a protective put strategy and to a CPPI strategy. The protective put strategy uses the put option with the same strike price as the 0.7 delta call option. We also evaluate using at the money put options, but the returns are even worse for the put strategy. The protective put strategy purchases one put option per unit of the index to fully insure against market declines below the strike price. On average, this cost is 5.7% of the portfolio each year.

Although put-call parity [$C + X/(1+r)^t = P + S$] where C is the call price, X is the strike price, P is the put price, and S is the stock price, suggests the B-S call option strategy is basically the same as the protective put strategy, there are key differences applied in this study as mentioned earlier. One key difference is the use of VBMFX, or a similar fund, instead of the risk-free rate (though the study does use 1-year and 10-year T-Bond returns to explore the robustness of the results). Another major difference is the amount in the call option is fixed at 5%, 10%, or 15%. No adjustment is made in response to changes in the underlying index prices. Thus, the results will not be the same as a standard synthetic put option strategy.

The CPPI strategy, like the call option strategy, also assumes positions in a risky and a “less risky” asset. Although CPPI returns are usually calculated using one risky and one risk-free asset, an adjustment is made here to directly compare with the PI call option strategies above. In this case, we use the VBMFX instead of a risk-free asset. The proportion in the risky asset at time $t$ is the max{min[(m($M_t$ - $F$),$M_t$],0}where $M_t$ is the value of the portfolio. The floor is set at a fixed 90% of the portfolio value at the beginning of each year and $m$ (the multiplier) is set at five. This means 50% of the portfolio is invested in the market at the beginning of each year. We don't consider buying on margin or short selling, so the maximum investment in the market is 100%.  Rebalancing occurs monthly so even if the market falls 20% over the month, the floor is not breached. Since a 20% loss over a single month has only occurred once post WWII, (21% in October 1987), this seems a reasonable position for even the very risk averse.

Since the goal of all PI type strategies is to avoid large losses, while still enjoying the expected gains of the stock market, we report minimum returns along with the Sortino ratio (Sortino and Price [1994]), which is a measure of semi-variance below some target return threshold. In this case, we use the VBMFX as the target return as an investor could elect to invest 100% in a relatively safe bond portfolio.  We also report Keating and Shadwick's [2002] Omega ratio using a threshold of zero.  It is similar to the Sortino ratio and both are subsets of Kappa, a more generalized risk-adjusted performance measure (Kaplan and Knowles [2004]).

For completeness, we report the Sharpe ratio even though it is not the best measure for assessing the downside risk PI portfolios are attempting to mitigate.  To determine whether PI Sharpe ratios are significantly better than buy-and-hold or vice versa, we use Opdyke's [2007] testing procedure.  This test accounts for the correlation between the series.

For robustness, we bootstrap 1990-2015 monthly returns to create 10,000 unique annual returns. Monthly returns are resampled with replacement for the S&P 500, VBMFX, one-year Treasury yield, 90-day Treasury yield, S&P 500 dividend yield, and the value of the Chicago

Board Option Exchange Volatility Index, (VIX). See Appendix 1 for more details about the B-S historical and simulated calculations.

With 10,000 observations, we employ additional metrics to measure portfolio performance. Along with the Sharpe, Sortino, and Omega ratios, we calculate value-at-risk (VaR), providing the percentage loss at some confidence level. We use a 5% confidence level, so VaR represents the percentage loss that will only be realized less than 5% of the time. To measure the magnitude of the loss below VaR, we calculate the expected shortfall (ES) to measure the average percentage loss below 5% (Acerbi and Tashe [2002].

To test for differences in VaR, we use an unconditional coverage test following Annaert et al. [2009]. We compare VaR values to the buy-and-hold VaR to determine whether the PI strategies have a significantly smaller VaR. With a 95% confidence level, under the null hypothesis that the buy-and-hold VaR equals the PI VaR, 5% of the PI observations should exceed the buy-and-hold VaR.

To determine whether the ES differences are significant, we follow Annaert et al. [2009]. From the 10,000 Monte Carlo runs, we sample 5,000 paired returns 10,000 times and recalculate ES for the buy-and-hold and PI strategy. We apply a paired sample t-test to the ES distributions created from the Monte Carlo simulations.

Although there is no official test to determine significant differences in Sortino or Omega ratios, this study modifies Scherer [2009] to create confidence intervals around the ratios to determine at what level of significance they overlap the buy-and-hold strategy. This involves bootstrapping the data 10,000 times to create a distribution of Sortino or Omega ratios. We create confidence intervals to determine whether the buy-and-hold Sortino or Omega ratio overlaps the PI confidence interval to create a 1% significance level. If the confidence intervals do not overlap, the ratios are significantly different. In addition, we apply the same procedure used to determine statistical differences in ES values to the Sortino and Omega ratios as a check on the results.

This study also determines whether the PI portfolios are superior to buy-and-hold based on Kahneman and Tversky's [1979] prospect theory which assumes investors have S-shaped utility functions where losses are given greater weight than similar gains. To place a prospect value on the portfolios, we use Tversky and Kahneman's [1992] value function and parameter estimates to calculate a cumulative prospect value. Since the original version of prospect theory has issues with potential violations of first order stochastic dominance, we calculate cumulative prospect values.

Following Tversky and Kahneman [1992], the probability weighting parameter for gains is 0.61 and 0.69 for losses. Dichtl and Drobetz [2011b] use a similar methodology in comparing dollar cost averaging to lump sum investing. Following their methodology, we use a paired t-test to determine whether there are significant differences in the cumulative prospect values.

## EMPIRICAL FINDINGS

### 13-month June or December Options

Exhibit 1 Panel A provides average annual returns from 1990-2014 for a buy-and-hold strategy, three B-S portfolios, a protective put, and CPPI strategy. The buy-and-hold represents a 100% investment in the S&P 500. The three B-S portfolios are based on investments ranging from 85%-95% in Vanguard's Investment grade bond fund (VBMFX), with the remaining percentage invested in 13-month 0.7 delta call options expiring in December of the following year. The protective put strategy uses the same strike price as the 0.7 delta call options, and the

CPPI monthly rebalancing strategy is based on a 90% floor, with 50% initially in the market while the remainder is in VBMFX.

PI strategies mitigate downside risk by giving up some upside. The goal of an effective PI strategy would be not to give up too much of the upside. Thus, it is somewhat surprising the 85/15 portfolio outperforms the buy-and-hold strategy and has an overall geometric average return of 11.77% compared to the S&P's 10.50%. Although not shown, this result remains for both the 1990-2001 and 2002-2014 subperiods, so it is not an artifact of just one time-period. The CPPI also performs well with a performance slightly better than the 95/5 and less than the 90/10. The protective put strategy appears to be suboptimal with an overall return of only 7.91%, by far the worst of the group. It performs poorly relative to the call option and CPPI strategies primarily due to the return benefits from the VBMFX.

**Exhibit 1**
**PI Strategy Comparisons.**

| | S&P | 85/15 | 90/10 | 95/5 | Protective Put | CPPI | VBMFX |
|---|---|---|---|---|---|---|---|
| Panel A: 13-Month December Options | | | | | | | |
| Arith. Avg. | 11.89% | 13.15% | 10.88% | 8.61% | 7.91% | 9.96% | 6.33% |
| Geo. Avg. | 10.50% | 11.77% | 10.18% | 8.34% | 7.41% | 9.55% | 6.23% |
| St. Dev. | 16.59% | 18.57% | 13.05% | 7.91% | 10.67% | 9.81% | 4.69% |
| Minimum | -37.59% | -14.26% | -10.50% | -6.73% | -10.67% | -8.43% | -2.97% |
| Sharpe | 0.44 | 0.46 | 0.54 | 0.65 | 0.40 | 0.65 | 0.65 |
| Sortino | 0.40 | 0.80 | 0.86 | 0.92 | 0.18 | 0.73 | N/A |
| Omega | 5.13 | 7.33 | 11.38 | 21.80 | 6.53 | 14.16 | N/A |
| Panel B: 13-Month June Options | | | | | | | |
| Arith. Avg. | 10.94% | 11.71% | 9.93% | 8.14% | 6.38% | 10.46% | 6.35% |
| Geo. Avg. | 9.76% | 10.45% | 9.34% | 7.95% | 5.87% | 10.08% | 6.27% |
| St. Dev. | 15.59% | 17.54% | 11.80% | 6.53% | 7.80% | 9.35% | 4.20% |
| Minimum | -31.87% | -10.28% | -6.26% | -2.77% | -5.55% | -4.63% | -0.59% |
| Sharpe | 0.43 | 0.42 | 0.53 | 0.75 | 0.39 | 0.75 | 0.77 |
| Sortino | 0.32 | 0.51 | 0.56 | 0.62 | -0.02 | 0.76 | N/A |
| Omega | 4.75 | 6.52 | 13.36 | 65.47 | 12.03 | 23.41 | N/A |

This exhibit shows the average annual returns for the S&P 500, three Strategies in which 85%-95% is invested in Vanguard's investment grade bond fund and the remaining is invested in 13-month June or December call options (delta 0.7), protective put strategy, and a CPPI strategy.

All the call option PI portfolios minimize downside risk, in some cases by very large amounts. The Sharpe, Sortino, and Omega ratios for all three call option strategies are greater than the buy-and-hold strategy with the Sortino ratios twice that of the S&P 500 demonstrating superior downside risk-return performance. This is further exemplified by the minimum annual returns for the PI call option portfolios, which have minimum returns of -14.26% for the 85/15 and as little as -6.73% for the 95/5 relative to S&P's -37.59%.

The protective put strategy has the worst overall return along with the worst Sharpe, Sortino, and Omega ratios. The CPPI strategy, excluding any transaction costs, performs well relative to the option strategies. Its geometric average return exceeds the 95/5 but trails the 85/15 and 90/10. However, the CPPI has a lower Sortino ratio for the overall period as well as both individual time periods.[3] The Omega ratio for the CPPI is greater than the 90/10, but much less than the 95/5.

As mentioned in the methodology, the value of selling the call before expiration is to avoid some of the significant time decay over the last month. This decay would only be significant when the call option is relatively close to being at the money. This occurred in Nov. 2000, and 2007 where the time value of the options was $15.13 and $16.16 respectively representing 8% to 20% of the original cost and recovering 10% to 20% of the original time premia. In all other years, the options were either so deep in or out of the money, there was little time value to recuperate. Thus, the value of selling with one month before expiration does not occur often, but with little cost for rolling over the position and occasionally recuperating value, it remains a viable although not critical strategy.

To determine whether the November to November results are unique, we examine the May to May time frame from 1990 to 2015 using 13-month June call options.[4] Exhibit 1 Panel B presents the results. The results are similar to Panel A with no major differences noted. Again, the Sortino and Omega ratios for all three strategies are superior to the S&P 500 without sacrificing returns.

Overall, the results suggest PI portfolios provide up to twice as much excess return for the downside risk. They reduce maximum losses dramatically and absolute returns are commensurate and even exceed a buy-and-hold strategy in some cases. One possible explanation is that the call option LEAPS may be undervalued relative to the benefits they can provide. The poor results for the protective put strategy certainly give some credence to Wiggins [2001], who has suggested that the market overprices put option LEAPS approximately 80% of the time. Several studies have suggested that index options are mispriced (Ackert and Tian [2001]; Constantinides, Jackwerth, and Perrakis [2009]; Chambers, Foy, Liebner, and Lu [2014], which may partially explain some of the results, although the returns to the VBMFX play a large role.

**Comparison of 0.5 and 0.7 call option deltas**

To test the robustness of the above results for the call option strategies, we examine options closest to 0.5 delta, which are generally at-the-money. Exhibit 2 reports the results for the 90/10 PI strategy. Although the returns are similar to using 0.7 delta call options, the results generally indicate more risk as measured by the standard deviation or minimum return and have lower Sharpe, Sortino, and Omega ratios. Although not shown, this held for the subperiods as well. We expect this result, since the time premium makes up the vast majority of the price of an at-the-money option. Overall, risk-return reward using these options is not as attractive compared to using the 0.7 delta call options.

**Exhibit 2**
*Option delta comparison.*

| | **Delta = 0.5** | **Delta = 0.7** |
|---|---|---|
| Arith. Avg. | 11.01% | 10.88% |
| Geo. Avg. | 9.88% | 10.18% |

| | | |
|---|---|---|
| St. Dev. | 17.03% | 13.05% |
| Minimum | -12.64% | -10.50% |
| Sharpe | 0.39 | 0.54 |
| Sortino | 0.69 | 0.86 |
| Omega | 9.71 | 11.38 |

This exhibit compares returns for the 90/10 Strategy Using Two Different delta Strategies from the End of the Month of November to November, 1990-2014.

### Combining June and December options

One of the drawbacks to initiating the position only once a year is the possibility of paying a greater than average price for the option if volatility happens to be particularly high at the time. To mitigate this issue, an investor could buy a 13-month June option in May and, six months later, buy the 13-month December option in November. In that way, an originally purchased 13-month option would be replaced each six months and the typical 90/10 strategy would be a 90/5/5. The only downside to this diversification strategy that provides a little bit of insurance against temporarily high volatilities is cost as there is an additional round-trip transaction in options (compared to a 13-month June or December strategy). Exhibit 3 presents the results for this type of portfolio.

**Exhibit 3**
*Combining June and December options.*

| | **S&P** | **85/7.5/.7.5** | **90/5/5** | **95/2.5/2.5** | **CPPI** | **VBMFX** |
|---|---|---|---|---|---|---|
| Arith. Avg. | 11.45% | 12.98% | 10.71% | 8.46% | 9.12% | 6.23% |
| Geo. Avg. | 10.41% | 11.92% | 10.20% | 8.27% | 8.84% | 6.14% |
| St. Dev. | 13.76% | 14.95% | 10.32% | 6.15% | 7.63% | 4.15% |
| Minimum | -34.78% | -9.91% | -6.33% | -3.90% | -8.25% | -3.04% |
| Sharpe | 0.53 | 0.59 | 0.69 | 0.84 | 0.75 | 0.73 |
| Sortino | 0.63 | 1.14 | 1.20 | 1.26 | 0.88 | N/A |
| Omega | 4.21 | 4.93 | 7.05 | 15.08 | 9.57 | N/A |

This exhibit shows annualized returns for the S&P 500 and three strategies in which 85 to 95% is invested in Vanguard's Investment grade bond fund and the remaining is invested in 13-month June and December call options (delta 0.7) along with a CPPI based on 6-month holding periods from 1990 to 2015. Note that the minimum return is the non-annualized six-month worst return.

The results in Exhibit 3 indicate that purchasing options every six months can provide higher returns, lower standard deviations, and better minimum returns than purchasing annual options. Purchasing options every six months also result in higher Sortino and Sharpe ratios relative to purchasing options only once a year. Again, the returns are commensurate with the S&P 500, while drastically reducing the minimum return. Thus, an investor who wishes to buy the long-term call options each May and November may be even better off than an investor who sets up this type of strategy just once a year, either in May or in November. We also implement a CPPI strategy where we change the fixed floor of 90% every six months. The CPPI strategy continues to have higher returns than the 95/5 strategy, but, in general, has Sharpe, Sortino, and Omega ratios between the 90/10 and 95/5.

**Option Length and Bond Fund**

To see how critical the maturity of the option is, we investigate returns for buying the 16-month June option in February where the option is then sold the following February. We also evaluate using the December option where a 16-month call is purchased in August. Results using the 16-month option are not significantly different from returns using 13-month options.

Another possible bias of the PI call option strategies is the selection of the bond fund used. Although this study uses the VBMFX since it has been around for 25 years, investing the other 90% in 10-year T-bonds works equally well, and for this time frame, the PI strategies perform even better when using 10-year T-bonds. Alternatively, a 1-year treasury could be used. This is the traditional "risk-free" asset used in a CPPI strategy. Exhibit 4 compares the returns, standard deviations, and minimums for the different bonds used.

**Exhibit 4**
*Return comparisons of using the 1-year Treasury, 10-year Treasury, and VBMFX.*

| | **85/15** | **90/10** | **95/5** | **CPPI** |
|---|---|---|---|---|
| | | 1-year Treasury | | |
| Avg. Return | 10.64% | 8.21% | 5.79% | 8.67% |
| St. Dev. | 17.77% | 12.04% | 6.44% | 9.79% |
| | | 10-year Treasury | | |
| Avg. Return | 13.83% | 11.59% | 9.36% | 10.50% |

| | | | | |
|---|---|---|---|---|
| St. Dev. | 18.74% | 13.50% | 8.99% | 9.13% |
| | | VBMFX | | |
| Avg. Return | 13.15% | 10.88% | 8.61% | 9.96% |
| St. Dev. | 18.57% | 13.05% | 7.91% | 9.81% |

This exhibit shows summary returns for the S&P 500 and three strategies in which 85 to 95% is invested in either 1-year Treasuries, 10-year Treasuries, or the VBMFX bond fund while the remaining is invested in 13-month December call options (delta 0.7) from end of month November to November 1990-2014. A CPPI strategy is included.

Although the 10-year Treasury gives the best results, it is also riskier than the Vanguard fund as it is a constant 10-year Treasury, while the VBMFX is a 30% mix of corporate bonds and 70% government bonds with short, medium, and long durations. This is evidenced by the higher standard deviations using the 10-year Treasury. Exhibit 4 also shows the return advantage from using a riskier fixed income investment in the 85/15 to 95/5 portfolios. The CPPI is also helped with the 10-year Treasury or VBMFX fund, but not to the degree the B-S portfolios are. This is because the CPPI only starts out with 50% in the bond fund, and may go to 100% in the risky asset. This cannot be the case for the B-S portfolios as a large percentage of their portfolios will always be in the bond fund. In conclusion, the B-S portfolios are helped by the higher fixed income returns relative to the CPPI. In addition, the initial conclusions remain about the value of a B-S strategy as Exhibit 4 shows the overall results are not an artifact of a particularly successful investment grade bond fund.

**Monte Carlo results**

As a final check on the robustness of the results demonstrated thus far in this study, we employ Monte Carlo simulation, in effect bootstrapping as we resample the 1990-2015 monthly data with replacement to create 10,000 unique annual returns. This allows a much better forecast of what might occur in the future and simultaneously eliminates any bias in the results due to the sequence of returns exhibited from 1990-2015. The 1990-2015 average monthly return is 0.86% and the monthly standard deviation is 4.2%. This is somewhat lower than historical data dating to 1926 which has an average monthly return of 0.94% and a monthly standard deviation of 5.51%, but shows the results generated will have the same basic return and volatility characteristics the market has experienced for the last 90 years.

As reported in Exhibit 5, the simulations confirm the earlier results as the arithmetic average return from the 85/15 is higher than the S&P 500 and the 90/10 is similar, while still reducing the minimum loss from -42.00% to -21.04% and -16.67%, respectively. There are no geometric means to report as the results are simply based on one-year simulations. The Sortino ratio using all 10,000 simulated years was 0.55 for the S&P 500 and a consistent 0.81 for all three call option PI portfolios where differences did not arise until several decimal points, if at all. This is the case since when one portfolio's return exceeded the VBMFX, all three did. Using zero as the target return led to a much greater discrepancy for the Sortino ratios, from 1.31 for the S&P to 1.67, 2.29, and 5.47, respectively, for the 85/15, 90/10, and 95/5 portfolios. All three Sortino ratios for the call option PI portfolios are significantly greater than the S&P 500. The Omega ratios reconfirm these results.

**Exhibit 5**
*Monte Carlo simulation results.*

| | S&P | 85/15 | 90/10 | 95/5 | Prot. Put | CPPI | VBMFX |
|---|---|---|---|---|---|---|---|
| Arith. Avg. | 10.89% | 12.38%*** | 10.32% | 8.26% | 7.46% | 9.29% | 6.20% |
| St. Dev. | 16.04% | 19.92% | 13.67% | 7.72% | 14.05% | 12.03% | 3.93% |
| Minimum | -42.00% | -21.04% | -16.67% | -12.29% | -34.11% | -14.61% | -9.19% |
| Sharpe | 0.47 | 0.45 | 0.51*** | 0.64*** | 0.29 | 0.50* | 0.73*** |
| Sortino (VBMFX) | 0.55 | 0.81*** | 0.81*** | 0.81*** | 0.15 | 0.59 | N/A |
| Sortino (0 floor) | 1.31 | 1.67** | 2.29*** | 5.47*** | 0.90 | 2.01*** | N/A |
| VaR | -14.54% | -11.80%*** | -6.98%*** | -2.63%*** | -11.59%*** | -5.77%*** | -0.32%*** |
| ES | -0.20 | -0.14*** | -0.09*** | -0.05*** | -0.17*** | -0.08** | -0.02*** |
| Omega | 5.86 | 6.18*** | 10.04*** | 26.17*** | 4.24 | 11.42*** | N/A |
| SD (1st and 2nd) | | No SD | No SD | No SD | No SD | No SD | No SD |
| CPV | 0.075 | 0.116*** | 0.114*** | 0.103*** | 0.052 | 0.107*** | 0.084 |

This exhibit shows the results for 10,000 runs in which 85 to 95% is invested in Vanguard's Investment grade bond fund and the remaining is invested in 13-month call options. The results report the yearly average returns.
*The test statistic is significant at the 10% level
**The test statistic is significant at the 5% level
***The test statistic is significant at the 1% level

The protective put strategy, once again, does poorly. Not only is the return much less than the other portfolios, the reduction in risk based on the Sharpe, Sortino, or Omega ratio is not apparent. The ratios are smaller than those for the buy-and-hold strategy. The CPPI appears to fall somewhere between the 95/5 and 90/10 strategy although all else being equal, the 90/10 appears better in terms of higher absolute return. The 90/10 also has higher Sharpe and Sortino ratios compared to the CPPI, while the VaR and ES are approximately equal. The Omega ratio for the CPPI is marginally larger than of the 90/10. All the PI strategies have better VaR and ES values than the buy-and-hold and significantly so with the exception of the protective put VaR value.

To the extent that prospect theory describes an average investor, we compute the cumulative prospect values (CPVs) for the PI strategies. The 85/15, 90/10, 95/5, and CPPI strategies have comparatively equal CPVs that are all significantly larger than the buy-and-hold strategy represented by the S&P 500. This suggests the average investor would likely prefer one of the PI strategies relative to a buy-and-hold strategy. Even the 95/5, which gives up a couple of return percentage points to the S&P 500, has a significantly larger CPV.

The final question may be which is better, the CPPI or the option based strategy. The 90/10 has a 1% greater annual return and a higher Sortino ratio, but also has a slightly higher standard deviation and a slightly lower VaR, ES, minimum return, and Omega ratio. The CPV and Sharpe ratios between the two are basically identical. Thus, the results are mixed between the two. Trading costs and ease of position creation would certainly favor the 90/10.

To compare the two more accurately, we vary the percentage invested in the call option so that the average return from the CPPI and the option based strategy are approximately equal. Exhibit 6 presents the resulting 92/8 portfolio compared to the CPPI. The risk metrics for the two strategies are inconclusive. The 92/8 has a slightly worse minimum and a lower Sortino ratio, but a higher Sharpe ratio, smaller VaR, smaller ES, and higher Omega values. The CPVs are also similar. Thus, this study finds option based portfolio insurance is certainly as attractive as the standard CPPI method and much easier to implement without necessary monthly ore even more frequent rebalancing.

**Exhibit 6**
*Setting B-S portfolio returns equal to CPPI or S&P.*

| | **CPPI** | **92/8** | **S&P** | **89/11** |
|---|---|---|---|---|
| Arith. Avg. | 9.29% | 9.31% | 10.89% | 10.90% |
| St. Dev. | 12.03% | 10.91% | 16.04% | 15.32% |
| Minimum | -14.61% | -16.02% | -42.00% | -18.02% |
| Sharpe | 0.50 | 0.55*** | 0.47 | 0.49 |
| Sortino (VBMFX) | 0.59 | 0.56 | 0.55 | 0.68*** |
| VaR | -5.77% | -5.23% | -14.54% | -8.27%** |
| ES | -8.07% | -7.38% | -20.35% | -10.35%*** |
| Omega | 11.42 | 13.64*** | 5.86 | 8.67*** |
| SD (1st and 2nd) | No SD | No SD | No SD | No SD |
| CPV | 0.107 | 0.113 | 0.075 | 0.115*** |

This exhibit shows results for 10,000 runs and compares the CPPI to the 92/8 and the S&P 500 to an 89/11. The results provide yearly average returns. Significance results based on 89/11 significantly better than the S&P 500 value or significant difference between the 92/8 and CPPI.
*The test statistic is significant at the 10% level
**The test statistic is significant at the 5% level
***The test statistic is significant at the 1% level

Furthermore, the option based strategy is much more flexible. Where the CPPI has an upper limit of 100% investment in stocks with no floor, we can adjust the option based model to equal market returns, at least on an expected return basis, and to exceed the expected market returns using the 85/15. To directly compare a 100% buy-and-hold S&P portfolio to the OBPI, we adjust the option percentage until the expected returns are equal. Using Monte Carlo simulation, we find an 89/11 portfolio provides the same average return as an investment in the S&P portfolio. Exhibit 6 also compares the 89/11 portfolio to the S&P portfolio.

With an expected return equal to the S&P, the 89/11 has a lower standard deviation, a minimum less than half of the S&P, and greater Sharpe, Sortino, and Omega ratios with a superior VaR and ES. Although there is not first or second order SD, the CPV is significantly greater, as well. Thus, the option based model put forth in this study is flexible enough to be equivalent to the CPPI and provides superior return-risk characteristics relative to a buy-and-hold strategy.

**No Black Swans**

As with any portfolio insurance strategy, there is a cost to insuring against downside risk. Cesari & Cremonini (2003) found CPPI strategies are dominant in bear and no-trend markets and it is important to determine when a B-S portfolio may do better or worse. Exhibit 7 shows the returns based on different market conditions by separating the simulated results into categories relative to both the equity and fixed income market.

**Exhibit 7**
*PI average returns under various market environments.*

| | **S&P** | **85/15** | **90/10** | **95/5** | **Prot. Put** | **CPPI** | **VBMFX** |
|---|---|---|---|---|---|---|---|
| S&P < -5% | -12.54% | -9.91% | -4.80% | 0.31% | -9.44% | -4.46% | 5.42%. |
| -5% to 5% | 0.41% | -2.79% | 0.18% | 3.15% | -3.29% | 0.70% | 6.12% |
| 5% to 10% | 7.51% | 5.65% | 5.80% | 5.95% | 2.87% | 4.77% | 6.09% |
| > 10% | 23.39% | 27.22% | 20.31% | 13.39% | 18.28% | 18.18% | 6.47% |
| VBMFX < 0% | 6.40% | 1.55% | 0.47% | -0.60% | 3.86% | 2.53% | -1.67% |
| 0% to 5% | 9.42% | 8.22% | 6.47% | 4.72% | 6.17% | 7.13% | 2.97% |
| 5% to 10% | 11.69% | 14.23% | 11.93% | 9.64% | 8.17% | 10.32% | 7.34% |
| > 10% | 13.14% | 19.37% | 16.95% | 14.53% | 9.33% | 12.99% | 12.11% |

This exhibit shows summary returns for the S&P 500 and the various PI strategies. Results are based on sorting 10,000 simulations by the S&P 500 and VBMFX bond return.

When the S&P is down 5% or more, all the PI portfolios outperform as would be expected. Although only the average returns are shown for conciseness, PI portfolio's primary advantage is their much smaller minimums in these periods. When the market is relatively flat, -5% to 5%, there is not a great deal of dispersion across the portfolios except for the 95/5 which tends to outperform in these conditions. During large positive market moves as represented by a market return of 10% or more, the PI portfolios tend to lag, but do participate a great deal on the upside. Interestingly enough, the 85/15 outperforms the S&P on average when it increases by more than 10% since its option position realizes very high returns.

When the fixed income market does poorly, the B-S portfolios all underperform the market as well as the CPPI. This is evidenced in Exhibit 7 when the VBMFX has a negative return for the year. This continues for the 90/10 and 95/5 even when the fixed income returns are positive but low. Above 5%, the 90/10 does significantly better than a CPPI and above 10%, even better. Thus, the options components of the B-S allow these portfolios to participate in much of the upside in the equity market, but tend to lag with low returns in the fixed income market.

These results are reaffirmed when looking at the empirical results since 2005. Exhibit 8 shows the results. All the PI strategies performed as they should have in 2008 by avoiding the large 37% drop in the S&P. Over the entire 10-year period, the S&P 500 underperformed the 85/15 by almost 1.5% annualized and was approximately equal to the 90/10 and CPPI strategy. On a yearly basis, the S&P 500 outperforms the 85/15 in 6 of 10 years, but its arithmetic average is less than the 85/15 and only 1.53% greater than the 90/10. The CPPI and 90/10 portfolios give very similar results with each outperforming the other 5 of 10 years.

However, with no black swan event by excluding 2008, along with excluding the very high returns in 2009 and 2013 to see what occurs on average during “normal” years, the S&P 500 had an average return of 11.63%. This was 2.13%, 3.6%, and 5.05% greater than the 85/15, 90/10, and 95/5 respectively.  The shows the cost of PI portfolios when the market is doing well relative to a buy-and-hold portfolio.  Despite this cost, with only one down year in the last 10, B-S portfolios demonstrate their long-run value with only the 95/5 lagging the S&P 500 while the 85/15 outperformed.

**Exhibit 8**
*PI Annual Returns from 2008 to 2014.*

| Date | S&P | 85/15 | 90/10 | 95/5 | Prot. Put | CPPI | VBMFX |
|---|---|---|---|---|---|---|---|
| 2005 | 8.78% | 4.21% | 3.58% | 2.95% | 4.68% | 6.04% | 2.32% |
| 2006 | 14.10% | 14.33% | 11.49% | 8.65% | 9.74% | 11.46% | 5.81% |
| 2007 | 7.86% | 2.88% | 3.98% | 5.08% | 4.02% | 5.28% | 6.18% |
| 2008 | -37.59% | -13.39% | -8.29% | -3.20% | -8.18% | -8.43% | 1.89% |
| 2009 | 25.61% | 16.60% | 14.84% | 13.08% | 12.18% | 17.78% | 11.32% |
| 2010 | 9.96% | 6.75% | 6.45% | 6.15% | 2.27% | 5.87% | 5.85% |
| 2011 | 7.64% | 4.46% | 4.70% | 4.95% | 0.54% | 2.20% | 5.19% |
| 2012 | 16.18% | 11.45% | 9.43% | 7.40% | 7.45% | 10.93% | 5.38% |
| 2013 | 30.22% | 33.36% | 21.58% | 9.80% | 23.08% | 21.71% | -1.98% |
| 2014 | 16.92% | 22.39% | 16.59% | 10.78% | 11.44% | 13.01% | 4.98% |
| Geo. Return | 8.19% | 9.64% | 8.14% | 6.47% | 6.43% | 8.28% | 4.64% |
| Avg. w/2008 | 9.97% | 10.31% | 8.44% | 6.56% | 6.72% | 8.58% | 4.69% |
| Avg. exc. 08,09,13 | 11.63% | 9.50% | 8.03% | 6.57% | 5.73% | 7.83% | 5.10% |

This exhibit shows the average annual returns for the S&P 500, three Strategies in which 85%-95% is invested in Vanguard's investment grade bond fund and the remaining is invested in 13-month December call options (delta 0.7), protective put strategy, and a CPPI strategy.

One concern when implementing PI programs is whether they will adjust quickly enough if the risky asset price has a sudden and significant price drop. This is referred to as gap risk. On this front, the B-S strategy is superior to CPPI as the call based PI strategy does not rely upon continual adjustment to a portfolio in response to market movements. In effect, using call option LEAPS allows investors the ability to avoid Black Swan events in their own portfolios with as little as two trades a year.  There is less gap risk with the B-S strategies since a significant percentage of the portfolio is always in a bond fund.

In contrast, a CPPI strategy will always face gap risk. The percentage in the risky asset for a CPPI strategy often reaches 100% in the risky asset. Even if rebalanced daily, a decline such as Oct. 19, 1987 would still see a 20% drop in the portfolio if the percentage in the risky asset had reached 100% beforehand. In this study, that would have been the case. With monthly rebalancing, even if a 20% loss is not realized in any given month, the CPPI strategy with a 90% floor can still lose more than 20% between the points in time that the floor value is reset. This is also true for the B-S portfolios, but the risk is mitigated as these portfolios will always have a significant percentage in the bond fund.

To alleviate the CPPI gap risk, an investor could move to daily rebalancing, but with up to 500 trades a year at $5 a trade, the transactions costs alone would be approximately 2.5% annually for a $100,000 portfolio. In this study, the cost of implementing the CPPI using monthly rebalancing would have been approximately 0.12% for a $100,000 portfolio based on 24 trades at $5 each. For a $10,000 portfolio, this cost is 1.2%. The B-S strategy has a great advantage for small investors, since the expense would only be approximately 0.2% for a $10,000 portfolio. This shows the additional flexibility of the B-S strategy, although an option on the SPY may have to replace buying an option on the index itself with such a small account.

**SUMMARY AND CONCLUSIONS**

Portfolio Insurance is based on the premise that investors don’t want full exposure to the market. They want to participate in the market’s upside potential, but don’t want to suffer the full consequences of declines in the market. They specifically desire to mitigate the impact of large market declines on the value of their portfolio. Prior research focuses primarily on put based OBPI and CPPI strategies to accomplish this goal. This study focuses on B-S portfolios that invest a fixed percentage in a bond fund, with the remainder invested in long-term call options on the S&P 500.

Results from this study suggest that B-S portfolios have superior return and downside risk characteristics. The 90/10 strategy, where 10% is invested in long-term call options, has returns commensurate with the S&P 500, while mitigating losses. An 85/15 strategy has returns that exceed the S&P 500, while still reducing maximum loss, and a 95/5 strategy reduces maximum loss to as little as 12%.

Compared to a buy-and-hold strategy, call option PI strategies (B-S portfolios) have significantly higher Sharpe, Sortino, and Omega ratios and better VaR and ES values. As shown by other researchers, the protective put strategy does poorly both in the empirical data and based on bootstrapped results, while the CPPI tends to have returns better than the 95/5, but worse than the 90/10 with not quite as attractive Sharpe or Sortino ratios. Changing the bond/option weight to a 92/8 equates the B-S average return to the CPPI average return. The risk metrics between the two strategies are virtually identical under this scenario. All the PI strategies have statistically greater cumulative prospect values suggesting an average investor may prefer these types of portfolios over a buy-and-hold strategy.

The PI call option strategies are robust to the delta of the option, the length of the option, and to the bond fund used. The results are affirmed using simulations. A PI call option strategy offers excellent downside risk protection, while maintaining much of the upside appreciation.

Investors wishing to reduce maximum loss exposure without giving up too much of the upside return should consider B-S portfolios. They may be more attractive to investors and financial planners due to the ability to tailor them to specific risk profiles and due to the ease of managing the positions.  The B-S portfolios require significantly fewer transactions and don’t require rebalancing until the end of the protection period.

**ENDNOTES**

[1] As popularized by Nassim Taleb after the 2008-2009 stock market crash, Black Swan events typify very rare occurrences that nevertheless have the potential to have a significant impact.

[2]While not integral to the current study, the 12-month holding period is established to ensure favorable, long-term tax treatment of any potential option gains. SPX index options have additional tax advantages as 60% of any realized gains from holding periods of less than 365 days is still treated as a capital gain.

[3]We ignore transaction costs in this study. The impact of that assumption is more severe for CPPI as that strategy involves monthly transactions. The B-S PI portfolios require only annual rebalancing and we did account for bid-ask spreads as the option is purchased at the ask and sold at the bid.

[4]Due to data availability, the last year in which the December strategy could be implemented is 2014. However, data availability for June options allows us to extend it to 2015.

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## Appendix 1

The equation to calculate the returns for the B-S portfolios is given by:

$$R_{BS} = x(C_{13} - C_1)/C_{13} + (1 - x)R_{VBMFX} \quad (1)$$

where $R_{BS}$ is the B-S portfolio return, x is 5%, 10%, or 15% for the 95/5, 90/10, and 85/15 B-S portfolios respectively, $C_{13}$ is the call option price with a 0.7 delta and 13 months to expiration based on the last day of November for an option expiring the following December in 13 months, $C_1$ is the last day of November call price for the option purchased 12 months earlier, and $R_{VBMFX}$ is the return to the bond fund from November to November based on the same dates as the call option. For the June options, replace the November dates with the month of May, and for the 16-month options, the call prices are based on 16 months and 4 months to maturity expiring in either December or June.

For the bootstrapped data, 1990-2015 monthly data is sampled with replacement to create 10,000 annual returns. Each month is independently sampled. However, for that month, the return for the VBMFX, one-year treasury, 90-day treasury, S&P 500 return, S&P 500 dividend yield, and CBOE's VIX index are concurrently sampled to maintain the relationship between these variables during any particular month. Option prices are then calculated based on Merton's (1973) constant dividend model given as:

$$C = Se^{-qt}\, N(d_1) - Xe^{-rt}\, N(d_2) \quad (2)$$

where $d_1 = \frac{Ln\left(\frac{S}{X}\right) + (r - q + \sigma^2/2)t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$

C is the call option price with 385 or 20 days to maturity, S is the S&P 500 value, q is the S&P 500 dividend yield, X is the strike price, and N() is the standard normal cumulative distribution function. X is set at 0.9356 times S which was the average amount the call options were in the money based on the empirical data for 0.7 delta options. Where the VIX is a near perfect estimate for the option when sold, it is used as a proxy for the 13-month option when bought. The average sampled VIX was 19.89% with a standard deviation of 7.59%. This should include most if not all implied volatilities that might be seen on a 13-month option. The average implied volatility for the November empirical data was 19.68% with a standard deviation of 5.14%. Thus, the use of VIX appears to be a reasonable proxy for what the implied volatility could be even for the 13-month options.